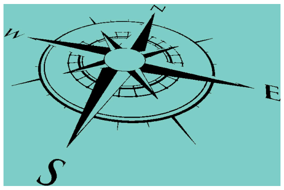
About Logic and Proof

[T]hese are the goals of this course:

- You should learn to write clear, “literate,” mathematical proofs.
- You should become comfortable with symbolic logic and the formal modeling of deductive proof.
- You should learn how to use an interactive proof assistant.
- You should understand how to use logic as a precise language for making claims about systems of objects and the relationships between them, and specifying certain states of affairs.

Let us take a moment to consider the relationship between some of these goals. It is important not to confuse the first three. We are dealing with three kinds of mathematical language: ordinary mathematical language, the symbolic representations of mathematical logic, and computational implementations in interactive proof assistants. These are very different things! Symbolic logic is not meant to replace ordinary mathematical language, and you should not use symbols like $\wedge$ and $\vee$ in ordinary mathematical proofs any more than you would use them in place of the words “and” and “or” in letters home to your parents. Natural languages provide nuances of expression that can convey levels of meaning and understanding that go beyond pattern matching to verify correctness. At the same time, modeling mathematical language with symbolic expressions provides a level of precision that makes it possible to turn mathematical language itself into an object of study. Each has its place, and we hope to get you to appreciate the value of each without confusing the two. The proof languages used by interactive theorem provers lie somewhere between the two extremes. On the one hand, they have to be specified with enough precision for a computer to process them and act appropriately; on the other hand, they aim to capture some of the higher-level nuances and features of informal language in a way that enables us to write more complex arguments and proofs. Rooted in symbolic logic and designed with ordinary mathematical language in mind, they aim to bridge the gap between the two.

Contents



Logic and Proof: Release 0.1 (L+P)

Description:

Both this online textbook and the Lean theorem prover it invokes are new and ongoing projects, and in places they are still rough. Please bear with us! You can learn more about Lean from its project page and the online textbook, Theorem Proving in Lean.

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1. Mathematical Proofs

Learn to write clear, "literate," mathematical proofs.

The first goal of this course is to teach you to write clear, readable mathematical proofs.

2. Symbolic Logic & Deductive Proof

Become comfortable with symbolic logic and the formal modeling of deductive proof.

The goal of symbolic logic is to identify [the] core elements of reasoning and argumentation and explain how they work, as well as to explain how more domain-specific notions are introduced and used... The second goal of this course is to help you understand natural deduction, as an example of a formal deductive system.

3. Proof Assistant

Learn how to use an interactive proof assistant.

A goal of the field of interactive theorem proving is to reach the point where any contemporary theorem can be verified ... The third goal of this course is to teach you to write elementary proofs in Lean.

4. Logical Expressions

Understand how to use logic as a precise language for making claims about systems of objects and the relationships between them, and specifying certain states of affairs.

... the goal of symbolic logic is to specify a language and rules of inference that enable us to get at the truth in a reliable way... The fourth goal of this course is to convey the semantic view of logic, and to lead you to understand how logical expressions can be used to specify states of affairs.